Method for measuring complex permeability at radio frequencies

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An established method for measuring complex rf magnetic permeability is based on the change in inductance and resistance of a coaxial transmission line upon insertion of a sample toroid. It is not necessary to wind coils on the toroid or correct for geometric demagnetization factors. The use of modern commercial impedance analyzers, as described in this paper, makes measurements from 1 kHz to 1 GHz particularly easy, fast, and accurate.

INTRODUCTION

In 1952, the National Bureau of Standards (NBS) *Technical News Bulletin* described two instruments built by P. H. Haas for use in the high-frequency calibration of magnetic materials.[1] The first was the NBS primary standard.

"The method of measurement depends on the change in inductance of an accurately machined [variable-length] coaxial line when a sample of magnetic material is inserted... . The output terminal of the coaxial line is connected to the "unknown" terminal of an rf bridge... . First, the bridge is balanced with no sample in the line... . The disk [toroid] of magnetic material is then placed on the center conductor of the line. A metal cap holding the disk in place short circuits the end of the coaxial line. The resultant bridge unbalance is adjusted to the original conditions by a combined manipulation of the resistance reading arm on the bridge and a reduction in the length of the line. The variation in length is directly proportional to the permeability of the magnetic material relative to air and constitutes a primary method of measurement. The difference in resistance readings is a measure of the loss factor of the material... ."

It was noted that "the use of such highly precise and delicate calibration equipment as part of a production line" was not practical. The second instrument described was the NBS rf permeameter, a secondary standard that used an impedance transformer. This instrument, based on an early design by Kelsall,[2] was extensively studied and developed in the following decade.[3-6] Both instruments are described in greater detail by Harrington.[7,8]

In 1962, Rasmussen and Powell published a note describing a low-impedance Maxwell bridge whose "unknown" arm consisted of a shorted section of coaxial line.[9] The inductance and resistance change due to insertion of a sample in this arm could be related to the complex permeability of the sample. This method was like the earlier NBS primary standard in that no impedance transformer was used. It was similar to the shorted-coaxial-line methods discussed by Epstein[10] in 1954 and by Miles *et al.*[11] in 1957. These articles, and those by Mulhall[12] and Bussey,[13] also review other permeability measurement methods at radio frequencies. Most of these methods were cumbersome and time consuming. The frequency range and accuracy of available bridges were limited.

In this paper we describe an easy, fast, and accurate method for measuring complex permeability as a function of frequency. It is based on the change in inductance ΔL and resistance ΔR of a shorted coaxial transmission line when a sample toroid is inserted, as in the method of Rasmussen and Powell.[9] Instead of a Maxwell bridge, a commercial impedance analyzer is used. The coaxial "line" is actually a 14-mm GR-900 precision connector with a removable shorting cap (Fig. 1).[14,15] A similar method, using a vector voltmeter equipped with a flexible probe, was used in the magnetic recording industry in the 1970's.[16] In 1984, Cagan and Guyot described a vector-voltmeter method that used a coaxial line and pick-up loops.[17]

Two improvements we present are the use of modern automated impedance analyzers, instead of impedance bridges or voltmeters, and the use of commercial coaxial connectors, instead of special lines or cavities. With these instruments we rapidly obtain plots of the real and imaginary components of permeability (μ' and μ'') at room temperature. Depending on the impedance analyzer used, the frequency may range from 1 kHz to 1 GHz. Below 1 kHz, the signal-to-noise ratio is unacceptably small.

A typical application of this type of permeameter might be the characterization of ferrites, tape-wound cores, or laminated amorphous alloys. Samples with high resistivity are required for complete field penetration and no significant skin-depth (eddy-current) effects. These materials are used

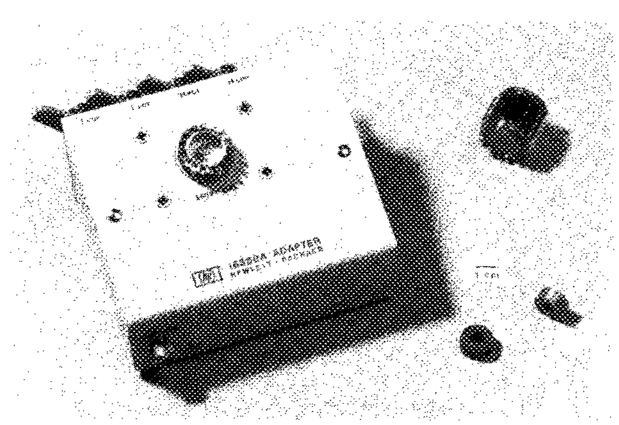

FIG. 1. Terminal adapter with 14-mm GR-900 precision coaxial connector, two sample toroids, and shorting cap. The terminal adapter is used with an impedance analyzer.

in magnetic recording heads, transformer cores, pulse-power cores, choke coils, saturable reactors, and magnetic amplifiers.

I. THEORY

Toroidal specimens are often used for the measurement of permeability of high-permeability materials owing to the absence of demagnetizing fields when the applied field is in the azimuthal direction. The typical method of achieving an azimuthal field is to wind coils on the toroid. This is tedious when it is necessary to measure many samples.[18] Furthermore, at frequencies above about 100 kHz, the distributed capacitance of the windings becomes significant.[10,11]

The TEM mode in a coaxial line has azimuthal magnetic field and radial electric field. The magnetic field has a maximum at a terminating short. A 14-mm GR-900 precision coaxial connector will accommodate toroids of rectangular cross section, 14.2-mm o.d., 6.3-mm i.d., and 7 mm in height. As will be shown, the samples do not need to completely fill the connector. They should, however, be positioned coaxially with the connector to ensure zero geometric demagnetization factor. The toroids may be easily extracted from the fixture using a bar magnet. Because of magnetic remanence in some ferrites, it is advisable to demagnetize the specimens with a decaying axial ac field before measurement.[19] Failure to demagnetize may result in measurement errors of 10% in some samples.

When there are losses in a material, the magnetic field and the flux density are out of phase by an angle δ. Thus both permeability μ and susceptibility χ are complex quantities. In SI units,

$$\mu \equiv \mu' - j\mu'' \equiv \mu_0(1+\chi) \equiv \mu_0(1+\chi' - j\chi'') , \quad (1)$$

$$\mu' = \mu_0(1+\chi') , \quad (2)$$

$$\mu'' = \mu_0 \chi'' , \quad (3)$$

where single primes indicate the real (dispersive, inductive) components, double primes indicate the imaginary (absorptive, resistive, loss) components, and μ_0 is the permeability of free space, $4\pi\times 10^{-7}$ H/m.

The inductance per unit length of a coaxial line is well known.[20] A toroid of inner diameter a, outer diameter b, height h, and real permeability μ', coaxial with the line, will contribute a material inductance equal to[21]

$$L_m = \mu' h \ln(b/a)/2\pi . \quad (4)$$

The inductance of the air space occupied by the toroid is

$$L_a = \mu_0 h \ln(b/a)/2\pi . \quad (5)$$

The change in inductance when the toroid is inserted in the coaxial sample holder (terminal adapter) is

$$\Delta L \equiv L_s - L_e = L_m - L_a , \quad (6)$$

where L_e is the measured inductance of the empty line and L_s is the measured inductance of the line with sample. Because inductors add in series, the inductance of the unoccupied portion of the coaxial line is removed by computing ΔL. Thus the toroid does not need to fill the line. Its dimensions, however, must be known. The relative real part of permeability is obtained by substituting Eqs. (4) and (5) in Eq. (6)

$$\mu_r' \equiv \mu'/\mu_0 = 1 + 2\pi\Delta L/[\mu_0 h \ln(b/a)] . \quad (7)$$

The same expression may be obtained from the relation[9,22,23]

$$\mu_r' = L_m/L_a = 1 + \Delta L/L_a . \quad (8)$$

In addition to ΔL, the core-loss term ΔR is measured. The definition of ΔR parallels that of ΔL in Eq. (6)

$$\Delta R \equiv R_s - R_e = R_m . \quad (9)$$

(The resistance R_a of the space to be occupied by the toroid is zero.) The imaginary part of permeability is[21,22]

$$\mu'' = \mu_0 R_m/(2\pi f L_a) , \quad (10)$$

where f is the frequency in Hz. Using Eqs. (5) and (9), we obtain the relative imaginary component of permeability

$$\mu_r'' \equiv \mu''/\mu_0 = \Delta R/[\mu_0 f h \ln(b/a)] . \quad (11)$$

The loss tangent (dissipation factor) is[22]

$$\tan\delta \equiv \mu''/\mu' = R_m/(2\pi f L_m) , \quad (12)$$

or equivalently,[9,23]

$$\tan\delta = \Delta R/[2\pi f(\Delta L + L_a)] = \Delta R(\mu_r' - 1)/(2\pi f \Delta L \mu_r') . \quad (13)$$

The quality factor Q is the reciprocal of $\tan\delta$. The relative loss factor is the quantity $\tan\delta/\mu_r'$.

II. EXPERIMENT AND RESULTS

The impedance analyzer is used initially to measure the inductance L_e and resistance R_e of the empty GR-900 connector and shorting cap as a function of frequency. The toroid sample is then inserted, the connector is shorted, and the inductance L_s and resistance R_s of the line with sample are measured versus frequency. Changes in series inductance ΔL and series resistance ΔR are computed. The real and imaginary components of relative permeability are calculated using Eqs. (7) and (11). Depending on the type of impedance analyzer used, the computation and conversion of ΔL and ΔR to μ_r' and μ_r'' may be done internally or with a computer.

A limited degree of controlled temperature variation is possible without any modifications to the terminal adapter shown in Fig. 1. A thermocouple may be attached to the outside of the shorting cap and warm or cool air flowed over the connector. An extension coaxial air line could be used for measurements in more extreme environments. The analyzer should be zeroed with the extra line in place. The use of BNC-type extension cables should be avoided.

Measurements of μ_r' and μ_r'' were made on eight commercial Mn–Zn and Ni–Zn ferrite toroids. Each frequency scan took 10 min. Values of μ_r' ranged from 12 to 12 000. Curves for one of them are shown in Fig. 2. The repeatability (precision) of the relative permeability measurements for a given impedance analyzer is estimated to be better than 1% for frequencies above 10 kHz. The μ_r' and μ_r'' results for all samples agreed with typical values provided by the manufacturer at least within 10%.

Measurements on four of the toroids (with μ_r' less than 200) were made using the NBS primary standard and an inductance bridge at 1 MHz.[24] The results for μ_r' agreed

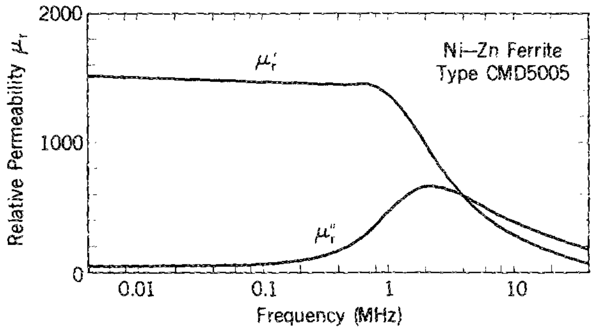

FIG. 2. Real and imaginary components of complex relative permeability (μ_r' and μ_r'') as functions of frequency for a typical Ni–Zn ferrite toroid sample. Each curve was measured as 200 discrete points, 256 averaged readings per point, 5-ms integration time per reading. The maximum frequency for the impedance analyzer used in the measurement was 40 MHz (see Ref. 14).

within 3%, which we estimate to be the systematic error (accuracy) of the impedance-analyzer method. A calibration measurement was made using a toroid of high-conductivity copper.[12] Both μ_r' and μ_r'' measured 0.00 ± 0.05 above 1 MHz, where the signal-to-noise ratio is high and the sample skin depth is small.

Normally, we use the maximum driving voltage of the impedance analyzer (1 V rms) to get the best signal-to-noise ratio. For the 40-MHz impedance analyzer,[14] this corresponds to a current of 20 mA rms and a magnetic field strength of 0.7 A/m (8 mOe) rms.[25] We compared measurements made at 1 V with some made at 0.5 V. Once again, they were in agreement within 1%. We can thus be confident that measurements at 1 V represent the *initial* permeability and that the loss characterized by μ'' represents resonance and relaxation, but not hysteresis.[10]

To verify that the method is independent of sample size, we machined two toroids of different dimensions from the same stock of sintered powdered Fe. One had about half the volume of the other. Measurements of μ_r' and μ_r'' for the two agreed within 1%. In general, the signal-to-noise ratio decreases at low frequencies and for small sample permeability and/or small sample size.

III. HIGH-FREQUENCY LIMITATIONS

There are theoretical and practical upper frequency limits to the measurement. The onset of the TE_{11} higher mode is at 9.5 GHz in the 14-mm GR-900 line. Below that, the support bead in the GR-900 connector may resonate at about 8.8 GHz.[26] For all frequencies, the distance between the sample and the short should be within a small fraction, typically 1%, of the free-space wavelength. This is because the magnetic field decreases as the cosine and the electric field increases as the sine as functions of distance from the short.

There are additional considerations in the case of materials with high μ_r' and/or high ϵ_r', where ϵ_r' is the real part of relative permitivity. The electrical length of the toroid is

$$\xi = h(\mu_r'\epsilon_r')^{1/2}, \tag{14}$$

if the toroid diameters are equal to those of the line. Because of the magnetostatic approximation implicit in the analysis, ξ should be small, less than about 5% of the free-space wavelength.[27] For example, a toroid with μ_r' and ϵ_r' both equal to 15 at 1 GHz should have h less than 1 mm. For most materials, μ_r' and ϵ_r' decrease sufficiently with increasing frequency such that the toroid can completely fill the connector and measurements be made up to 1 GHz with no problem.

If the toroid diameters are different from those of the line, μ_r' and ϵ_r' in Eq. (14) should be adjusted to smaller effective values that take into account the surrounding air space. Thus, an alternate way to decrease ξ is to reduce the toroid width $(b - a)$. Finally, the criterion of small ξ will raise the frequency for higher-mode self-resonance that is excited by eccentricity in the toroid, similar to the bead resonance in the coaxial connector.[26]

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Certain commercial materials and instruments are identified to specify the experimental study adequately. In no case does such identification imply recommendation or endorsement by the National Bureau of Standards, nor does it imply that the materials or instruments are necessarily the best available for the purpose.

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[8] The variable-length coaxial line may be modified for use as a half-wave resonant cavity at frequencies near 100 MHz. See Ref. 7 and C. A. Hoer and R. D. Harrington, "Parallel reversible permeability measurement techniques from 50 kc/s to 3 Gc/s," J. Res. NBS **67C**, 259 (1963).

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[13]H. E. Bussey, "Measurement of rf properties of materials, a survey," Proc. IEEE **55**, 1046 (1967); **56**, 729 (1968).
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[15]H. E. Bussey, National Bureau of Standards, Boulder, CO, used a GR-900 connector with Maxwell and Schering bridges for toroid permeability measurements in 1969.
[16]R. F. M. Thornley, Storage Technology Corporation, Louisville, CO (personal communication, 1986).
[17]V. Cagan and M. Guyot, "Fast and convenient technique for broadband measurements of the complex initial permeability of ferrimagnets," IEEE Trans. Magn. **MAG-20**, 1732 (1984).
[18]Convenient demountable coils are described in R. D. Harrington and A. L. Rasmussen, "Magnetic core permeability measurement techniques," in *Magnetic Core Conference Proceedings 7* (Magnetic Powder Core Association, New York, 1965), pp. 11–24. Contact resistance may be a problem with these coils.
[19]The initial permeability may depend on the method of demagnetization. See G. W. Rathenau and J. F. Fast, "Initial permeabilities of sintered ferrites," Physica **21**, 964 (1955). According to these authors, "the largest values [of permeability] are apparently obtained if the directions of measurement and demagnetization coincide." When the directions do not coincide, their data show a decrease in permeability of 10% in a Ni–Zn ferrite.
[20]S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics* (Wiley, New York, 1965), p. 304.
[21]R. I. Sarbacher and W. A. Edson, *Hyper and Ultrahigh Frequency Engineering* (Wiley, New York, 1943), p. 282. Their Eq. 7.172 for impedance in terms of complex permeability, $R_m + j\omega L_m = j\omega\mu \ln(b/a)/2\pi$, may be separated into real and imaginary components.
[22]A. L. Rasmussen and C. M. Allred, "An admittance meter technique to measure the complex permeability at VHF," J. Res. NBS **72C**, 81 (1968).
[23]A. L. Rasmussen and A. E. Hess, "R-F permeameter techniques for testing ferrite cores," Electr. Manuf. **61**, 86 (1958).
[24]Let $\Delta\lambda$ be the change in length of the line required to null the bridge after insertion of the sample. The inner and outer diameters of the line are A and B. The change in inductance corresponding to $\Delta\lambda$ is $\Delta L = \mu_0 \Delta\lambda \ln(B/A)/2\pi$. From Eqs. (4)–(6), $\mu_r' = 1 + \Delta\lambda \ln(B/A)/[h \ln(b/a)]$. In measuring μ_r'', defined in Eq. (11), we determine R_e by rebalancing the bridge *after* the line has been shortened and the sample removed. The variable-length line is not a primary standard for μ_r''.
[25]The azimuthal magnetic field strength as a function of current I and radial distance r in the GR-900 coaxial connector is $I/(2\pi r)$. For $I = 20$ mA, it ranges from 0.4 A/m (6 mOe) to 1 A/m (13 mOe), the same as the radial variation in field in a wire-wound toroid. The average magnetic field strength in the connector, obtained by integration, is $I \ln(B/A)/\{\pi(B - A)\}$, where A and B are the inner and outer diameters of the connector.
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[27]The criterion is $\tan(\pi\xi/\lambda)/(\pi\xi/\lambda) \simeq 1$, where λ is the wavelength in free space. See, for example, D. Polder, "Ferrite materials," Proc. Inst. Electr. Eng. (London) **97**, Part II, 246 (1950).